

Characterization of a Novel Magnetic Tracking System

Jason T. Sherman¹, Jonathan K. Lubkert², Radivoje S. Popovic³, and Mark R. DiSilvestro¹

¹DePuy Orthopaedics, Inc., Warsaw, IN 46581 USA

²Purdue University, West Lafayette, IN 47907 USA

³Microsystems Institute, EPFL, Lausanne CH-1015, Switzerland

A tracking system for determining the five-degrees-of-freedom position and orientation of a cylindrical permanent magnet was developed. The system consists of a planar array of 27 Hall effect magnetic sensors, interface electronics, and computer. A magnet is modeled by an ideal magnetic dipole. The closed-form solution for the magnetic flux density of the magnetic dipole was used with a Levenberg–Marquardt optimization algorithm to determine the coordinates of the dipole. The system was characterized using Nd–Fe–B magnets ($n = 5$, $B_r = 14.25 \pm 0.25$ kG, 2.00-mm diameter D , and 5.00-mm length L and $n = 5$, $B_r = 13.1 \pm 0.1$ kG, 4.55-mm D , and 6.35-mm L). The axis of each magnet was first aligned perpendicular to the center of the sensor array, and an automated positioner moved the magnet away from the array perpendicular to its face. Voltage data was collected from the 27 sensors at each 1-mm interval along this direction out to 150 mm for the smaller magnets and 300 mm for the larger magnets. Sensor offset and environmental magnetic flux density were measured after each test. A coordinate measurement machine verified the position and orientation of the magnets. A useful range of 100 mm from the array was determined for all magnets tested. Within this range, the position of smaller magnets along the axis of motion were determined with maximum error of 1.8 mm, while their position in the two orthogonal directions perpendicular to the axis of motion were determined with maximum error of 5.3 mm; the position of larger magnets along the axis of motion were determined with maximum error of 0.8 mm, while their position in the two orthogonal directions were determined with maximum error of 3.8 mm; and the maximum error in orientation for all magnets tested was 4.2° .

Index Terms—Hall effect, permanent magnets, position measurement, transducers.

I. INTRODUCTION

THIS paper investigated the utility of a novel magnetic sensor array comprised of Hall effect sensors to track the five-degree-of-freedom position and orientation of cylindrical permanent magnets. The concept of tracking magnets, either permanent or quasi-static, has been investigated by a number of authors over the last 25 years [1]–[6]. These investigators have shown utility in a number of industrial and biomedical applications. Those applications have described systems that were effective at position and orientation tracking of magnets of volumes greater than 75 mm^3 . The intended use of this system is to identify the position of implanted medical devices embedded in a patient's bone. This application requires minimal bone loss; therefore, magnets of smaller dimensions must be tracked than previously investigated. The overall size of the sensor array was a constraint, as it is intended to be part of a handheld device used by a surgeon intraoperatively. It was the objective of this work to design a system that could track magnets of appropriate volume within a range of 100 mm with accuracies on the order of 1 mm for position and 1° for orientation.

II. MATERIALS AND METHODS

A. Test Configuration

1) *Hardware*: System characterization used Nd–Fe–B magnets ($n = 5$, $B_r = 14.25 \pm 0.25$ kG, 2.00-mm diameter D , and 5.00-mm length L from Adams Magnetics, Elmhurst, IL and $n = 5$, $B_r = 13.1 \pm 0.1$ kG, 4.55-mm D , and 6.35-mm L from K&J Magnetics, Jamison, PA). The sensor array (Senis

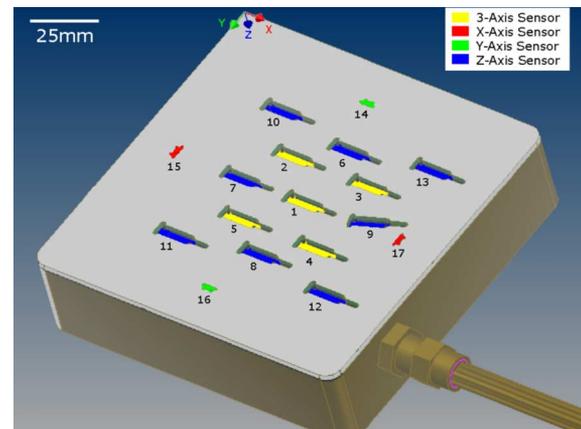


Fig. 1. Sensor array showing the respective locations of 3-D sensors (all others are 1-D) and sensors sensitive to magnetic flux density in each direction along with the orthogonal directions. The global coordinate system was simply translated to align the z -axis so that it passed through probe 1.

GmbH, Zurich, Switzerland) included five 3-D Hall effect sensors (C-H3A-2m_E3C–1 kHz–2.5–0.02 T) and 12 1-D Hall effect sensors (C-HYA-2m_E3S–100 Hz–2.5–2 mT) [7], [8]. The ranges for the 1-D and 3-D sensors were ± 2 and ± 20 mT, respectively; all sensors were verified to an accuracy of 0.25% of their respective full-scale range at room temperature. The sensors were mounted in a plastic (FR4) box in a planar configuration (Fig. 1). Sensors were connected to power and signal conditioning and were wired to a 12-bit PCI-6033E data acquisition card (National Instruments, Austin, TX). Signal conditioners wired to each sensor contain an analog 25-Hz low-pass filter.

A three-axes positioning stage, constructed from MA4000 and MA6000 UniSlide assemblies, (Velmex, Bloomfield, NY) was mounted to a nonmagnetic, level vibration isolation table (VERE, New Kensington, PA). The accuracy specification of

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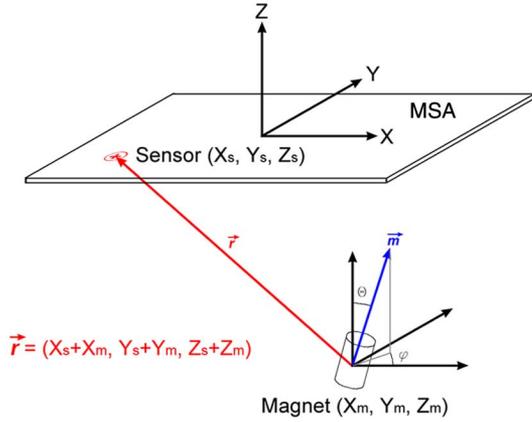


Fig. 2. Location of a given sensing element in the magnetic sensor array (MSA) is defined as (X_s, Y_s, Z_s) in Cartesian coordinates. The location of the magnet in the global coordinate system is (X_m, Y_m, Z_m) . The vector representing the distance from the magnet to the sensing element, (x, y, z) in (1), is $(X_s + X_m, Y_s + Y_m, Z_s + Z_m)$.

this device was less than $20 \mu\text{m}$ along a single axis. Custom fixtures were designed to hold the sensor array in a rigid location fixed to the table and to enable mounting of test magnets to the positioning stage. A nonmagnetic portable coordinate measuring machine (CMM) was used in this study to measure the true position and orientation of test magnets (Stinger II 4018 with a 2-mm D ruby tip, Romer, Wixom, MI). The point repeatability specification was $40 \mu\text{m}$.

B. Algorithm and Implementation

1) *Theory of Magnetic Tracking*: An equivalent ideal magnetic dipole model was used to model the magnet (1), [9]. In this model, magnetic flux density B is a function of the permeability of free-space ($\mu_0 = 4\pi \cdot 10^{-7}$ H/m), the orthogonal components of a vector distance from the magnetic dipole to a point in space ($x, y,$ and z in meters), the magnitude of this vector distance (r in meters), the magnet's magnetic moment (m in $\text{A} \cdot \text{m}^2$), the rotation of the magnetic moment from the positive z -axis towards the XY -plane (θ in $^\circ$), and the rotation in the XY -plane from the positive x -axis in counterclockwise direction (φ in $^\circ$) (Fig. 2). Each sensor measured a component of B ($B_x, B_y,$ or B_z)

$$B = \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \frac{\mu_0 m}{4\pi r^3} \left(3 \begin{bmatrix} x \\ y \\ z \end{bmatrix} \frac{x \sin \theta \cos \varphi + y \sin \theta \sin \varphi + z \cos \theta}{r^2} - \begin{bmatrix} \sin \theta \cos \varphi \\ \sin \theta \sin \varphi \\ \cos \theta \end{bmatrix} \right). \quad (1)$$

2) *Optimization Algorithm*: A Levenberg–Marquardt algorithm was implemented in MatLab 7.0 to determine the five parameters for each magnet tested at each point on the test vector. The objective function minimized the weighted sum of the squares of the differences between each measured component of B and the corresponding theoretical value. The inverse of the sensitivity of each sensor was used to weight its contribution to the objective function. The weighting factors were 2×10^{-5} (3-D) and 2×10^{-6} (1-D).

An initial guess for z was based on B_z measured; all other parameters were initially set to 0. A two-stage algorithm was devised that first fixed θ and φ and solved for $x, y,$ and z , and then fixed $x, y,$ and z and solved for θ and φ . The parameter bounds for x and y were ± 100 mm; the bounds for z were -200 mm and 0 m; the bounds for θ were $\pm 90^\circ$; and the bounds for φ were $\pm 180^\circ$. Convergence was limited by the default tolerance and a maximum of 1000 iterations.

C. Test Protocol

After preparing the fixture and mounting a test magnet, a global coordinate system was established with probe 1 set as the origin using the CMM. A local coordinate system was created on the fixture holding the test magnet, and magnet position and orientation were determined in the local system.

1) *Magnet Alignment*: Custom software written in LabVIEW 7.1 was used to align the axis of each test magnet perpendicular to the plane of the sensor array such that the axis of the magnet passed through probe 1. When B_x and B_y measured at probe 1 were both less than $15 \mu\text{T}$, the magnet was aligned.

2) *Measurements*: The local coordinate system and the magnet were reestablished with the CMM. The magnet was moved away from the sensor array in steps of 1.0 mm along the z -axis out to 15 cm (small magnets) and 30 cm (larger magnets). At each measurement point, 1500 voltage data samples were taken at 4 kHz from each of the sensors. The data was digitally filtered using a low-pass Butterworth filter with a cutoff frequency of 100 Hz. Custom software written in LabVIEW 7.1 was used to control the positioner and acquire data. The position and orientation of the magnet were found and the test path followed by the positioner was verified with the CMM. The magnet was removed from the fixture and voltage data was acquired from all sensors. This measured combined sensor offset and environmental magnetic flux density at that point in time, and was subtracted from each measurement point under the assumption that offset and environmental magnetic flux density remained constant through the duration of the test. All voltage data was converted to magnetic flux density.

3) *Magnetic Moment Determination*: Magnetic moments were calculated for each magnet using measurements made between 30 and 65 mm from the sensor array. This range was sufficiently far from the magnet for the dipole approximation to be valid, and excluded points below the noise floor of probe 1. The magnetic moment was calculated as the $2\pi/\mu_0$ times the slope of a line fit through the average B_z (T) measured by probe 1 at each point along the z -axis versus $1/z_{\text{meas}}^3$ (z_{meas} verified distance in meters).

4) *Magnet Position and Orientation Determination*: The magnetic moment and measured magnetic flux density components for each test magnet were input into the algorithm. The theoretical positions and orientations of each magnet at each point along the test vector were calculated $(x, y, z, \theta, \varphi)$.

III. TEST RESULTS

A. Magnetic Moment Determination

Magnetic moments for the small magnets were found to be $0.0173 \pm 0.0003 \text{ A} \cdot \text{m}^2$, while magnetic moments for the larger

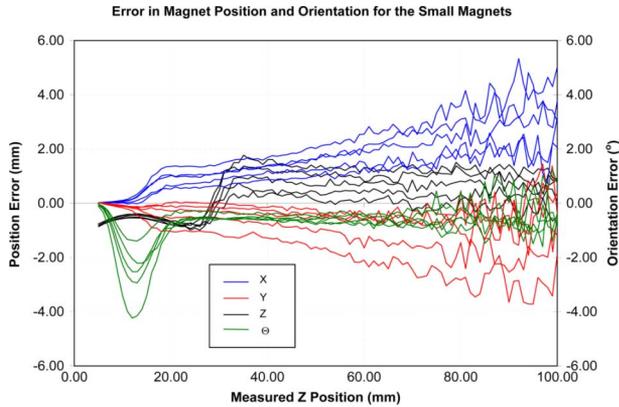


Fig. 3. Error in magnet position and orientation as a function of measured z position for the small magnets.

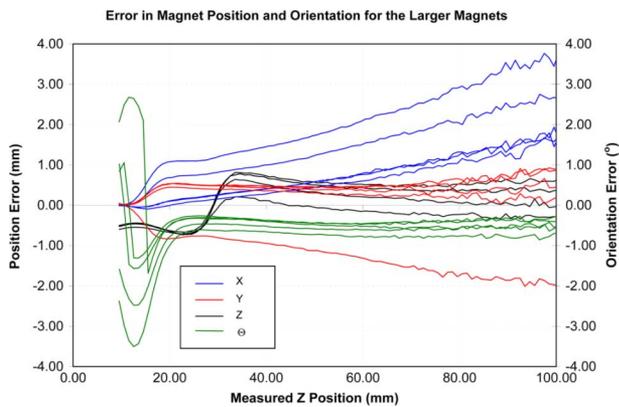


Fig. 4. Error in magnet position and orientation as a function of measured z position for the larger magnets.

magnets were found to be $0.111 \pm 0.001 \text{ A} \cdot \text{m}^2$. R^2 values for all linear fits exceeded 0.99.

B. Magnet Position and Orientation Determination

Fig. 3 depicts the error in magnet position and orientation for all five small magnets. Note that the error in x and y increased with measured z distance, while the error in z was steady out to 100 mm. The error in θ was largest near a measured z distance of 10 mm, and was otherwise steady out to 100 mm. Also note that results for φ are not shown as for angles of θ less than 5° (all tests), any value of φ between -180° and 180° has essentially no effect on magnet orientation.

Fig. 4 shows the error in position and orientation for all five larger magnets. Although all trends were consistent with those depicted in Fig. 1 for the small magnets, the magnitude of the error for all larger magnet parameters was always less than that of the respective small magnet parameters.

Table I lists the upper and lower bounds for error observed in x , y , z , and θ for both groups of magnets tested.

IV. DISCUSSION

Experimental errors contributing to the overall error seen included the accuracy of the positioner, CMM and sensor measurements, system noise, possible temperature variations in sensor sensitivities, and environmental/offset compensation. Although the ultimate goal of 1 mm and 1° accuracy was not

TABLE I
MAXIMUM ERROR BOUNDS FOR POSITION AND ORIENTATION PARAMETERS

Parameter	Small Magnets		Large Magnets	
	Lower Bound	Upper Bound	Lower Bound	Upper Bound
x (mm)	0.0	5.3	-0.1	3.8
y (mm)	-3.7	1.4	-2.0	0.9
z (mm)	-1.0	1.8	-0.7	0.8
θ ($^\circ$)	-4.2	1.3	-3.5	2.7

These bounds represent the error in parameters determined within the measured z range out to 100mm.

achieved, results from the first prototype were encouraging. Future improvements in accuracy will be investigated by including real-time environmental magnetic field compensation and more accurate measurements of sensor offsets taken inside a magnetically shielded area.

The system performance was only characterized along the z -axis. A more robust study would characterize the complete 3-D system performance. However, the technology is intended to be coupled with a targeting algorithm that would ensure proper initial alignment ($x, y, \theta, \varphi = 0$) along the z -axis before determining the exact z position of the magnet. The system is most accurate in determining the z position over the useful range. Future research will also focus on the development of an accurate and suitable targeting algorithm for the intended application.

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